Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY
Winter Examination-2018

Subject Name: Numerical Methods

Subject Code: 4SC04NUM1 **Branch: B.Sc. (Physics)**

Semester: 4 **Date : 29/10/2018** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:
(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Find the value of $\int_0^1 e^x \, dx$ with $h = 1/2$ by Trapezoidal rule. (02)

b) Give value of a & b such that root of $f(x) = 0$ lies between a & b,where $f(x) = x^2 + x - 5$. (02)

c) Give general formula for Modified Euler Method. (02)

d) The _____ method has a fast rate of convergence. (01)
(a) Bisection method (b) False position method
(c) Newton Raphson method (d) none of these

e) The modified Euler's method is the Runge-Kutta method of _____ order. (01)
(a) 3^{rd} (b) 1^{st} (c) 4^{th} (d) 2^{nd}

f) Out of four Runge-Kutta methods, the Runge-Kutta method of ___ order is most commonly used in applications. (01)
(a) 3^{rd} (b) 1^{st} (c) 4^{th} (d) 2^{nd}

g) Write Picard's formula for $\frac{dy}{dx} = f(x, y)$ with $f(x_0) = y_0$. (01)

h) What is the value of $f'(x)$ in general by Stirling inter polation formula? (01)

i) Write Simpson's one third rule. (01)

j) Write n^{th} approximation of iteration method. (01)

k) Give value of $f'''(x)$ by Newton's forward interpolation formula. (01)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a. Find the positive root of $2x = 3 + \cos x$ by bisection method. **(05)**

b. Find a root of $f(x) = 3x - 6 - \log_{10} x$ using Iteration Method up to four decimal places. **(05)**

c. Find a root of the equation $x^3 - 2x - 5 = 0$ correct up to three significant figures by using the Newton-Raphson method. **(04)**

Q-3 **Attempt all questions** **(14)**

a. Prove that Newton-Raphson Method has second order convergence. **(05)**

b. Apply Taylor's series method to obtain approximate value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$. **(05)**

c. Find a root of the equation cos x – xe^x = 0 correct up to three decimal places by using the False-position method. **(04)**

Q-4 **Attempt all questions** **(14)**

a. Find a root of the equation cos x – xe^x = 0 correct up to three decimal places by using the False-position method. **(05)**

b. Evaluate $\int_{0.1}^{0.7} e^x + 2x \, dx$ by Trapezoidal rule and taking n = 6. **(05)**

c. Evaluate $\int_{0}^{\frac{\pi}{2}} e^{\sin x} \, dx$ by Simpson's 3/8 rule and taking n = 6. **(04)**

Q-5 **Attempt all questions** **(14)**

a. Find $f'(0)$ from the following data: **(07)**

x	3	5	11	27	34
$f(x)$	-13	23	899	17315	25606

b. Derive $f'(x)$ by Newton's Forward Interpolation Formula. **(07)**

Q-6 **Attempt all questions** **(14)**

a. Determine $y(0.1)$ and $y(0.2)$ correct to four decimal places from $\frac{dy}{dx} = 2x + y, y(0) = 1$. Use fourth order Runge-Kutta method. **(07)**

b. Using Euler modified method, obtain a solution of $\frac{dy}{dx} = x + |\sqrt{x}|, y(0) = 1$ for the range $0 \leq x \leq 0.6$ in steps of 0.2 . **(07)**

Q-7 **Attempt all questions** **(14)**

a. Find the value of $f'(0.5)$ and $f''(0.5)$ using Stirling's formula from the following data **(07)**

x	0.35	0.40	0.45	0.50	0.55	0.60	0.65
y	1.521	1.506	1.488	1.467	1.444	1.418	1.389

b. The table given below reveals the velocity v of a body during the time t specified. Find its acceleration at $t = 1.1$ **(07)**

t	1.0	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.4	60.8

Q-8 **Attempt all questions** **(14)**

a. Find the solution of $\frac{dy}{dx} = e^x - y$ up to the fifth approximation. Using Picard's method given that $y(0) = 0$. **(07)**

b. Apply Milne's method to find the solution of the differential equation $\frac{dy}{dx} = x - y^2$ in the range $0 \leq x \leq 1$. **(07)**